TERRAMETRA

GRAPHS and FUNCTIONS
RECTANGULAR COORDINTES

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2.1
RECTANGULAR COORDINATES

- Ordered Pairs
- The Rectangular Coordinate System
- The Distance Formula
- The Midpoint Formula
- Equations in Two Variables

ORDERED PAIRS

ORDERED PAIRS

An ***<u>ordered pair</u>*** consists of two components, written inside parentheses.

The first component is the ***<u>independent</u>*** component.

The second component is the ***<u>dependent</u>*** component.

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Example 1
Ordered Pairs

1(a) Use the table to write ordered pairs to express the relationship between x and y.

X	Y
A	27
B	8
C	1
D	0
E	–1
F	–4
G	–9

Solution:

First row: (A , 27)

Third row: (C , 1)

Fifth row: (E , –1)

Last row: (G , –9)

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The Rectangular Coordinate System

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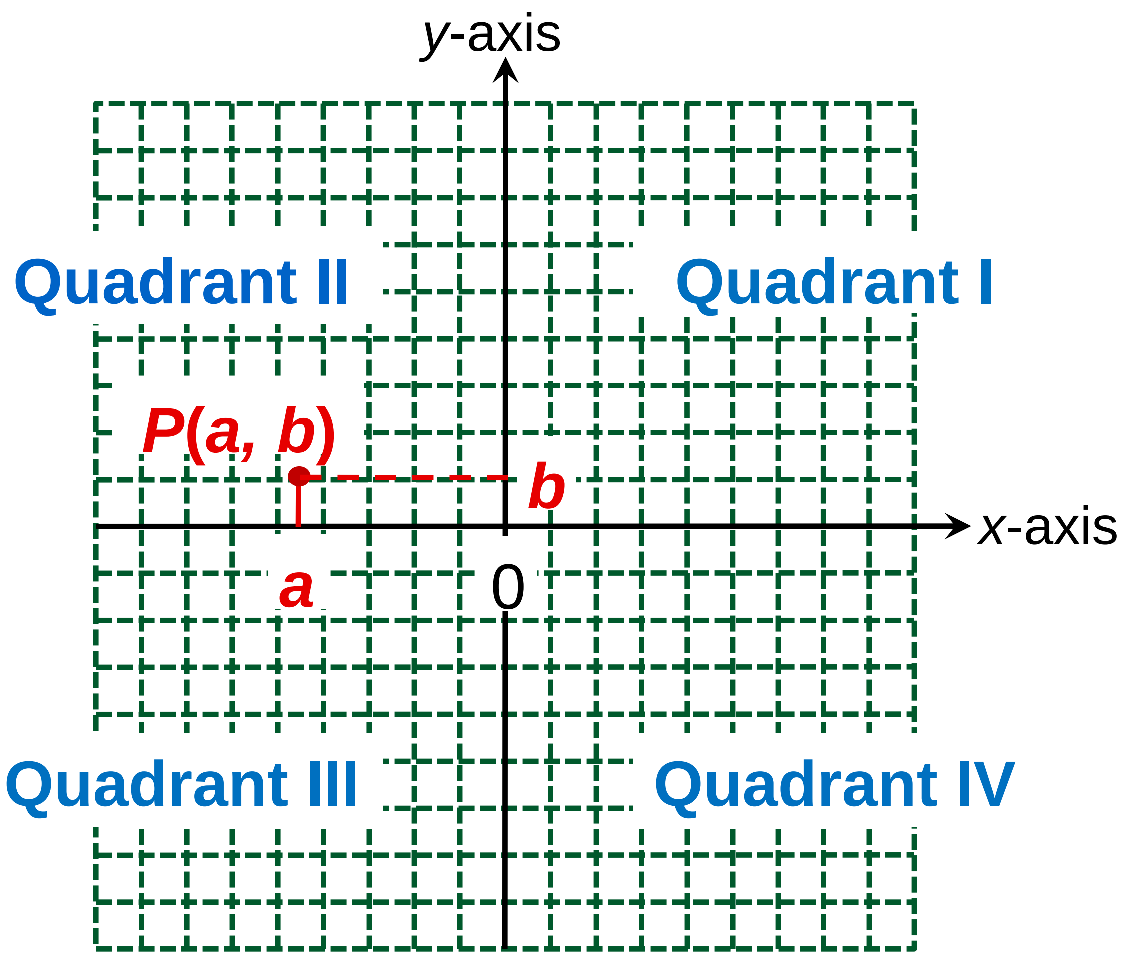
The Rectangular Coordinate System



PYTHAGOREAN THEOREM

PYTHAGOREAN THEOREM

Three points form a ***right triangle***,
if the lengths of the sides $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ satisfy

$$\boldsymbol{a^2 + b^2 = c^2},$$

where $\boldsymbol{c}$ (the longest side) is the hypotenuse,
and $\boldsymbol{a}$ and $\boldsymbol{b}$ are the legs of the triangle.

DISTANCE FORMULA

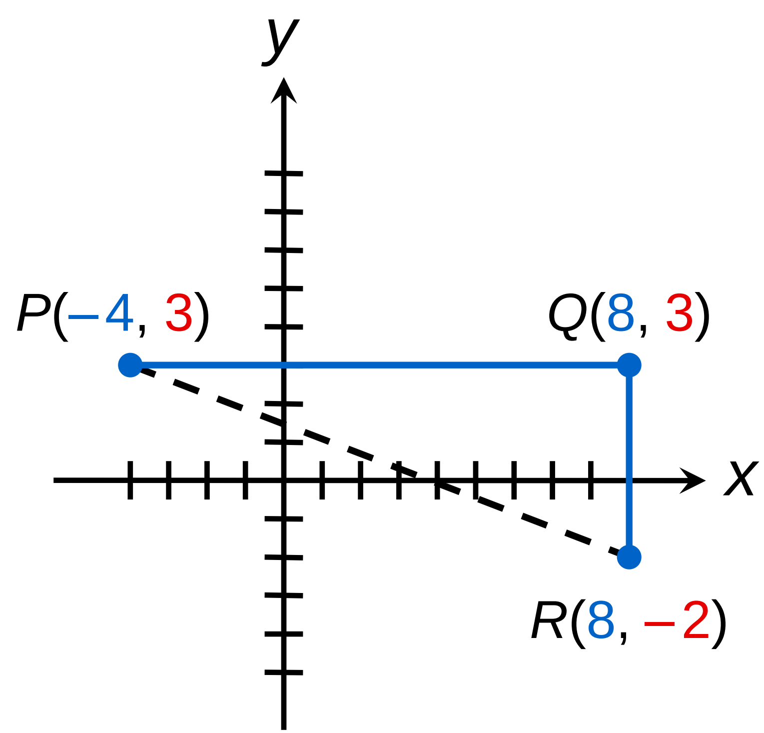
Using the coordinates of ordered pairs,
we can find the distance between any two points in a plane.

The horizontal side of the triangle has length …

$$d(P, Q) = |8 - (-4)| = 12$$

The vertical side of the triangle has length …

$$d(P, Q) = |3 - (-2)| = 5$$



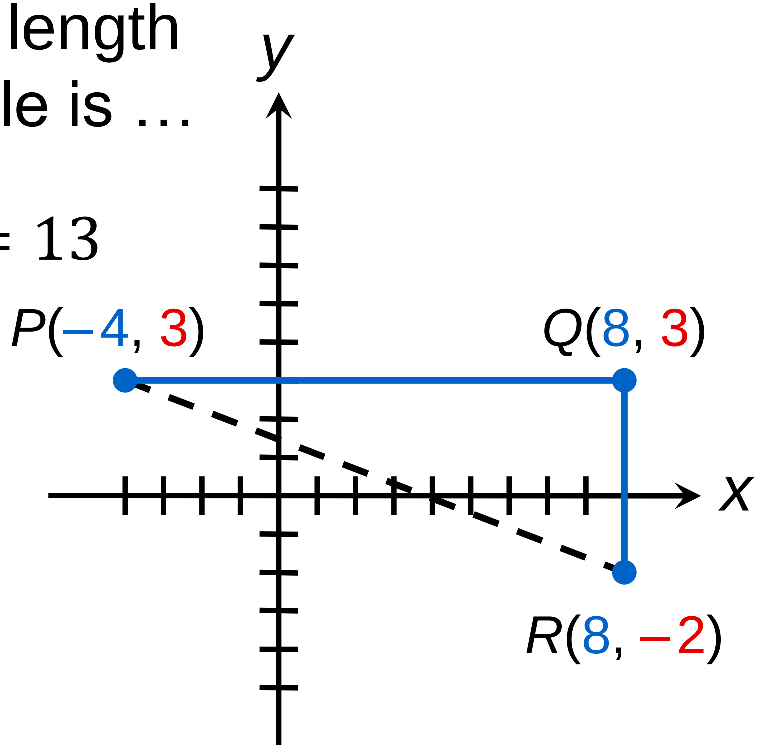
DISTANCE FORMULA

Using the coordinates of ordered pairs,
we can find the distance between any two points in a plane.

By the Pythagorean theorem, the length of the remaining side of the triangle is …

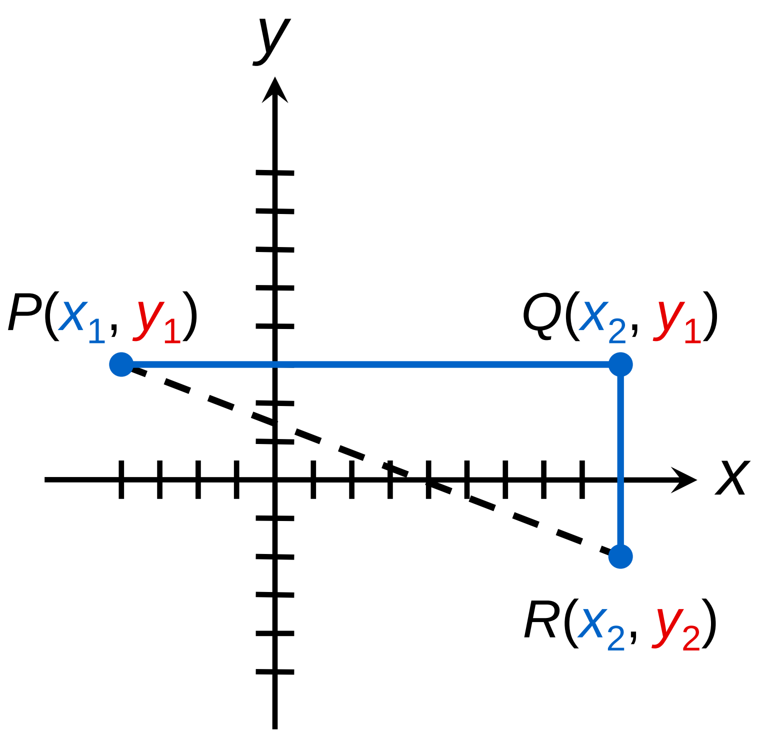
$$\sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

… so the distance between (– 4, 3) and (8, – 2) is 13.



DISTANCE FORMULA

To obtain a general formula for the distance between two points in a coordinate plane, let $P(x_1, y_1)$ and $R(x_2, y_2)$ be any two distinct points in a plane.



Complete a triangle by locating point Q with coordinates (x_2, y_1).

The Pythagorean theorem gives the distance between P and R …

$$d(P,R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

DISTANCE FORMULA

DISTANCE FORMULA

Suppose that $P(x_1, y_1)$ and $R(x_2, y_2)$ are two points in a coordinate plane.

The distance between P and R, written $d(P, R)$, is given by the following formula …

$$d(P,R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 2

Using the Distance Formula

2(a) Find the distance between *P*(– 8, 4) and *Q*(3, – 2).

Solution:

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[3 - (-8)]^2 + (-2 - 4)^2}$$

$$= \sqrt{11^2 + (-6)^2}$$

$$= \sqrt{121 + 36}$$

$$= \sqrt{157}$$

Be careful when subtracting a negative number.

Example 3

Using the Distance Formula

3(a) Determine whether the points *M*(– 2, 5), *N*(12, 3), and *Q*(10, – 11) are the vertices of a right triangle.

Solution:

$$d(M,N) = \sqrt{[12-(-2)]^2 + (3-5)^2}$$

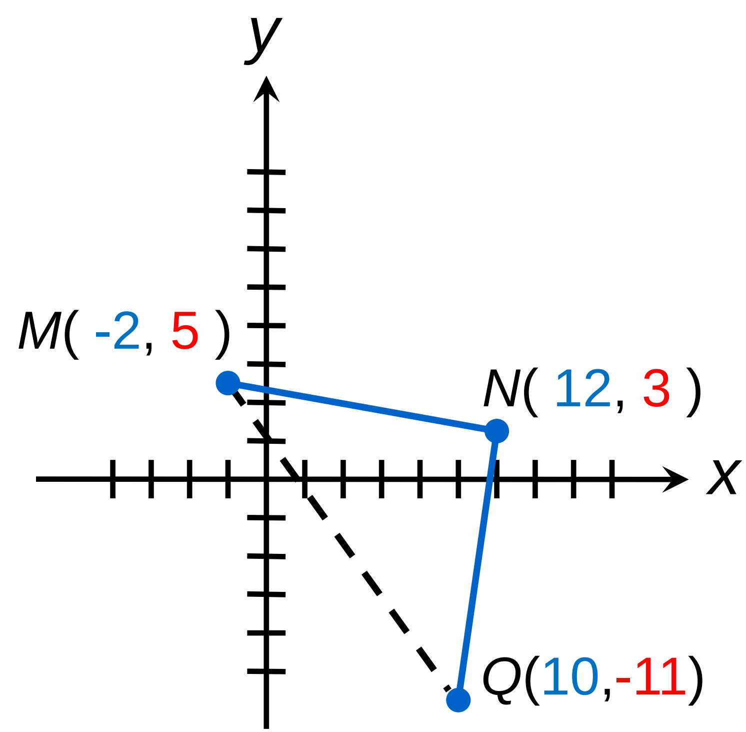
$$= \sqrt{196+4} = \sqrt{200}$$

$$d(M,Q) = \sqrt{[10-(-2)]^2 + (-11-5)^2}$$

$$= \sqrt{144+256} = \sqrt{400} = 20$$

$$d(N,Q) = \sqrt{(10-12)^2 + (-11-3)^2}$$

$$= \sqrt{4+196} = \sqrt{200}$$



Using the Distance Formula

3(a) Determine whether the points *M*(– 2, 5), *N*(12, 3), and *Q*(10, – 11) are the vertices of a right triangle?

Solution (cont'd):

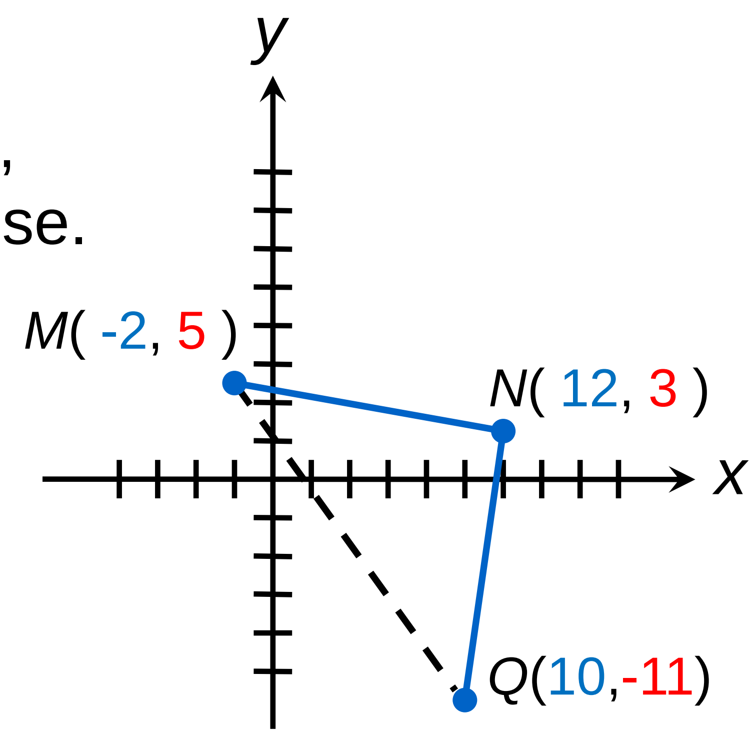
The longest side, of length 20 units, is chosen as the possible hypotenuse.

Since …

$$\left(\sqrt{200}\right)^2 + \left(\sqrt{200}\right)^2 = 400 = 20^2$$

… is true,

the triangle is a right triangle with the hypotenuse joining *M* and *Q.*



COLINEAR POINTS

COLINEAR POINTS

We can tell if three points are ***colinear***, that is, if they lie on a straight line, using a similar procedure.

Three points are colinear if the sum of the distances between two pairs of points is equal to the distance between the remaining pair of points.

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Example 4

Using the Distance Formula

4(a) Determine whether the points *P*(– 1, 5), *Q*(2, – 4), and *R*(4, – 10) are colinear.

Solution:

$$d(P,Q) = \sqrt{(-1-2)^2 + [5-(-4)]^2} = \sqrt{9+81} = \sqrt{90}$$
$$= 3\sqrt{10}$$

$$d(Q,R) = \sqrt{(2-4)^2 + [-4-(-10)]^2} = \sqrt{4+36} = \sqrt{40}$$
$$= 2\sqrt{10}$$

$$d(P,R) = \sqrt{(-1-4)^2 + [5-(-10)]^2} = \sqrt{25+225} = \sqrt{250}$$
$$= 5\sqrt{10}$$

MIDPOINT FORMULA

MIDPOINT FORMULA

The coordinates of the midpoint $M(x, y)$
of the line segment with endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$
are given by the following …

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

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Example 5

Using the Midpoint Formula

5(a) Use the midpoint formula to find the coordinates of the midpoint *M* of the line segment with endpoints (8, – 4) and (– 6,1).

Solution:

$$\left(\frac{8+(-6)}{2},\frac{-4+1}{2}\right)=\left(1,-\frac{3}{2}\right)$$

Substitute in the midpoint formula.

Example 5

Using the Midpoint Formula

5(b) Use the midpoint formula to find the coordinates of the other endpoint *Q* of a segment with one endpoint *P*(– 6, 12) and midpoint *M*(8, – 2).

Solution: Let (x, y) represent the coordinates of *Q*. Use the midpoint formula twice.

x-value of *P* x-value of *M*

$$\frac{x + (-6)}{2} = 8$$

Substitute carefully.

$$x - 6 = 16$$

$$x = 22$$

y-value of *P* y-value of *M*

$$\frac{y + 12}{2} = -2$$

$$y + 12 = -4$$

$$y = -16$$

The coordinates of endpoint *Q* are (22, – 16).

Using the Midpoint Formula

6(a) Find at least three ordered pairs that are solutions of the equation: $y = 4x - 1$

Solution: Choose any real number for x or y and substitute in the equation to get the corresponding value of the other variable.

$y = 4x - 1$		$y = 4x - 1$	
$y = 4(-2) - 1$	Let $x = -2$.	$3 = 4x - 1$	Let $y = 3$.
$y = -8 - 1$	Multiply.	$4 = 4x$	Add 1.
$y = -9$	Simplify.	$1 = x$	Divide by 4.

This gives the ordered pairs (– 2, – 9) and (1, 3).
Verify that the ordered pair (0, –1) is also a solution.

Example 6

Finding Ordered-Pair Solutions of Equations

6(b) Find at least three ordered pairs that are solutions of the equation: $x = \sqrt{y-1}$

Solution:

$1 = \sqrt{y-1}$ Let $\boldsymbol{x = 1}$.

$1 = y - 1$ Square each side.

$2 = y$ Add 1.

One ordered pair is (1, 2).

Verify that the ordered pairs (0, 1) and (2, 5) are also solutions of the equation.

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Example 6

Finding Ordered-Pair Solutions of Equations

6(c) Find at least three ordered pairs that are solutions of the equation: $y = x^2 - 4$

Solution:

A table provides an organized method for determining ordered pairs.

X	Y
–2	0
–1	–3
0	–4
1	–3
2	0

Five ordered pairs are …
(–2, 0), (–1, –3), (0, –4), (1, –3), and (2, 0).

Graphing an Equation by Point Plotting

Graphing an Equation by Point Plotting

Step 1 Find the intercepts.

Step 2 Find as many additional ordered pairs as needed.

Step 3 Plot the ordered pairs from Steps 1 and 2.

Step 4 Join the points from Step 3 with a smooth line or curve.

Example 7

Graphing Equations

7(a) Graph the equation: $y = 4x - 1$

Solution:

Step 1 Let $y = 0$ to find the x-intercept, and
Let $x = 0$ to find the y-intercept.

$y = 4x - 1$	$y = 4x - 1$
$0 = 4x - 1$	$y = 4(0) - 1$
$1 = 4x$	$y = 0 - 1$
$\frac{1}{4} = x$	$y = -1$

The intercepts are $\left(\frac{1}{4}, 0\right)$ and $(0, -1)$.

Example 7

Graphing Equations

7(a) Graph the equation: $y = 4x - 1$

Solution (cont'd):

Step 2 Find some other ordered pairs. (also found in **Example 5a**).

$y = 4x - 1$		$y = 4x - 1$	
$y = 4(-2) - 1$	Let $\boldsymbol{x = -2}$.	$3 = 4x - 1$	Let $\boldsymbol{y = 3}$.
$y = -8 - 1$	Multiply.	$4 = 4x$	Add 1.
$y = -9$	Simplify.	$1 = x$	Divide by 4.

This gives the ordered pairs (– 2, – 9) and (1, 3).

Example 7

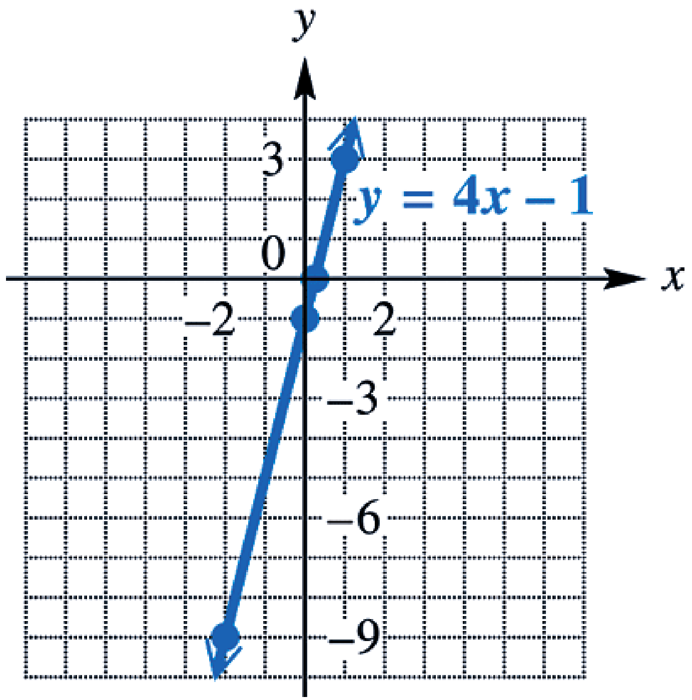
Graphing Equations

7(a) Graph the equation: $y = 4x - 1$

Solution (cont'd):

Step 3 Plot the four ordered pairs from Steps 1 and 2. (also found in **Example 5a**).

Step 4 Join the points with a straight line.



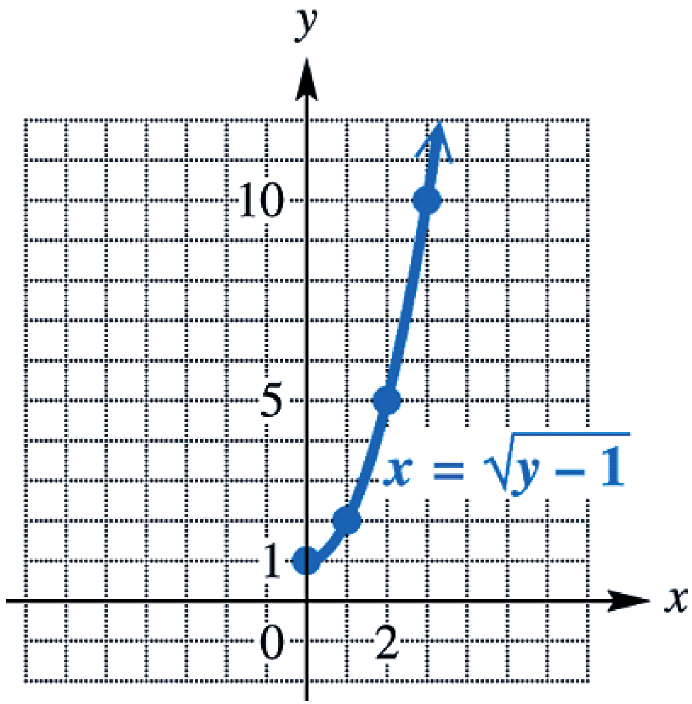
Example 7

Graphing Equations

7(b) Graph the equation: $x = \sqrt{y - 1}$

Solution:

Plot the ordered pairs found in **Example 5b**, and then join the points with a smooth curve. To confirm the direction the curve will take as x increases, find another solution, (3, 10).



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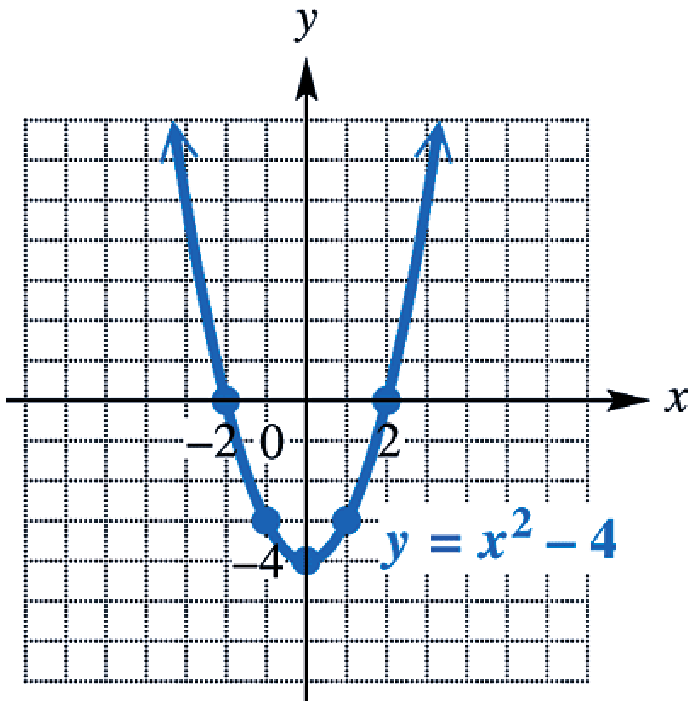
Example 7
Graphing Equations

7(c) Graph the equation: $y = x^2 - 4$

Solution:

Plot the ordered pairs and join them with a smooth curve.

X	Y
–2	0
–1	–3
0	–4
1	–3
2	0



This curve is called a **parabola.**